Vježbe 2

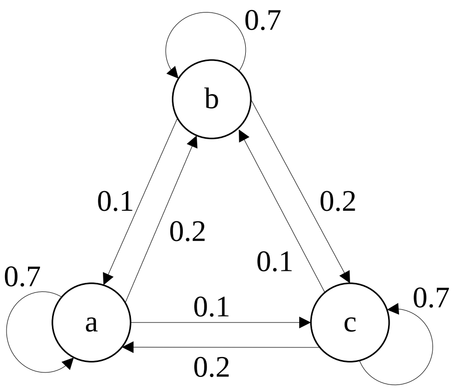
1. Dat je Markovljev izvor I reda sa alfabetom X={a,b,c} i uslovnim vjerovatnoćama datim tabelom:

$$\begin{bmatrix} 0.7 & 0.1 & 0.2 \\ 0.2 & 0.7 & 0.1 \\ 0.1 & 0.2 & 0.7 \end{bmatrix}$$

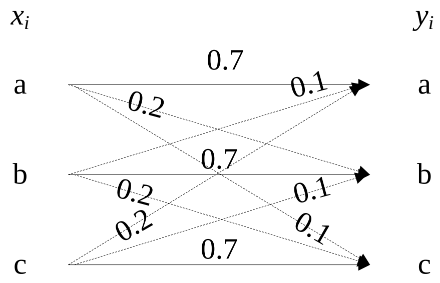
   a) Prikazati graf ovog sistema;
   b) Prikazati trelis ovog sistema;
   c) Prikazati vjerovatnoće simbola u stacionarnom stanju.

Rješenje:

a)



b)



c)

$$\begin{bmatrix} p'(a) \\ p'(b) \\ p'(c) \end{bmatrix} = \begin{bmatrix} 0.7 & 0.1 & 0.2 \\ 0.2 & 0.7 & 0.1 \\ 0.1 & 0.2 & 0.7 \end{bmatrix} \begin{bmatrix} p'(a) \\ p'(b) \\ p'(c) \end{bmatrix}$$

$$0.7p'(a) + 0.1p'(b) + 0.2p'(c) = p'(a)$$
$$0.2p'(a) + 0.7p'(b) + 0.1p'(c) = p'(b)$$
$$0.1p'(a) + 0.2p'(b) + 0.7p'(c) = p'(c)$$

$$-0.3p'(a) + 0.1p'(b) + 0.2p'(c) = 0$$
$$0.2p'(a) - 0.3p'(b) + 0.1p'(c) = 0 \quad / \times(-2)$$
$$0.1p'(a) + 0.2p'(b) - 0.3p'(c) = 0$$

Sabiranjem prve i druge jednačine dobijamo:

$$-0.7p'(a) + 0.7p'(b) = 0 \quad \Rightarrow \quad p'(a) = p'(b)$$

Iz treće jednačine dobijamo:

$0.3p'(c) = 0.1p'(a) + 0.2p'(b) \quad \Rightarrow \quad p'(c) = p'(a) \quad \Rightarrow \quad p'(a) = p'(b) = p'(c)$

Imajući u vidu da je:

$p'(a) + p'(b) + p'(c) = 1$

lako zaključujemo da je:

$$p'(a) = p'(b) = p'(c) = \frac{1}{3}$$

2. Pretpostaviti da su vjerovatnoće pojavljivanja simbola u prethodnom zadatku $p(a)=p(b)=2p(c)$.
   a) Odrediti vjerovatnoće pojave pojedinih simbola na izlazu iz kanala,
   b) Odrediti entropije ulaznog i izlaznog skupa simbola, H(X) i H(Y).

Rješenje:

a) Vjerovatnoće na ulazu su:

$p(a) = p(b) = 2p(c)$

$p(a) + p(b) + p(c) = 1$

$5p(c) = 1 \Rightarrow p(c) = 0.2$

$\Rightarrow p(a) = p(b) = 0.4$

Imajuću u vidu da je:

$$\begin{bmatrix} 0.7 & 0.1 & 0.2 \\ 0.2 & 0.7 & 0.1 \\ 0.1 & 0.2 & 0.7 \end{bmatrix} = \begin{bmatrix} p(y_1|x_1) & p(y_1|x_2) & p(y_1|x_3) \\ p(y_2|x_1) & p(y_2|x_2) & p(y_2|x_3) \\ p(y_3|x_1) & p(y_3|x_2) & p(y_3|x_3) \end{bmatrix}$$

možemo računati:

$$[p(x_i, y_j)] = [p(x_i)p(y_j|x_i)] = \begin{bmatrix} p(x_1)p(y_1|x_1) & p(x_1)p(y_2|x_1) & p(x_1)p(y_3|x_1) \\ p(x_2)p(y_1|x_2) & p(x_2)p(y_2|x_2) & p(x_2)p(y_3|x_2) \\ p(x_3)p(y_1|x_3) & p(x_3)p(y_2|x_3) & p(x_3)p(y_3|x_3) \end{bmatrix} =$$

$$= \begin{bmatrix} 0.4\times0.7 & 0.4\times0.2 & 0.4\times0.1 \\ 0.4\times0.1 & 0.4\times0.7 & 0.4\times0.2 \\ 0.2\times0.2 & 0.2\times0.1 & 0.2\times0.7 \end{bmatrix} = \begin{bmatrix} 0.28 & 0.08 & 0.04 \\ 0.04 & 0.28 & 0.08 \\ 0.04 & 0.02 & 0.14 \end{bmatrix}$$

Pošto je:

$$p(y_j) = \sum_{i=1}^{N} p(x_i)p(y_j|x_i),$$

to su vjerovatnoće na izlazu: $p(y_1)=0.36$; $p(y_2)=0.38$ i $p(y_3)=0.26$;

b) $$H(X) = -\sum_{i=1}^{N} p(x_i)\log p(x_i) = -[0.4\times\log 0.4 + 0.4\times\log 0.4 + 0.2\times\log 0.2] = 1.52 bit/simbol$$

$$H(Y) = -\sum_{i=1}^{N} p(y_i)\log p(y_i) = -[0.36\times\log 0.36 + 0.38\times\log 0.38 + 0.26\times\log 0.26] = 1.56 bit/simbol$$